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ON THE RTU VALUE

ROBERT J. JENNISON

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by

Robert J. Aumann

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# ON THE NTU VALUE

by

Robert J. Aumann\*

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A few year ago, A. Roth [1980] and W. Shafer [1980] published some examples in which, they argued, the Harsanyi-Shapley-Nash non-transferable utility value (Shapley [1969]) henceforth the NTU value looks strange and counter-intuitive. These articles were interesting because they shed light on this concept, at a time when it was finding an increasing number of applications. They also made it clear that the NTU value is in this respect similar to most economic and game-theoretic "solution concepts," almost all of which are beset by such examples. Just as the work of Roth and Shafer helped put the NTU value in perspective, perhaps the time has now come to put their work, in turn, in its proper perspective.

Section 1 is devoted to a selection of counterintuitive examples associated with several of the most traditional and respected solution concepts of game theory and economics; Next, we discuss how one can continue to live, and even thrive, in such an atmosphere; Section 2 discusses generalities, and Section 3 zeros in on the value. Finally, in Section 4, we return to a specific discussion of the above-mentioned examples; on closer examination, they are not as compelling as appears at first. A few concluding remarks constitute Section 5.

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1. There is a temptation to think of a solution concept in game theory like one thinks of a theorem in mathematics; one counter-example is enough to destroy a theorem, so perhaps one intuitively puzzling example should be enough to make one abandon a solution concept. But not much of game theory, nor indeed of economics, would survive such a doctrine. Let's start with the Nash equilibrium, the game theoretic concept that is perhaps best known and most frequently applied in economics. There are very simple, natural non-zero-sum two-person games (e.g., see Figure 1) that have a unique Nash

Figure 1

3,0	0,3
2,2	3,0

equilibrium point  $\sigma = (\sigma_1, \sigma_2)$ , which yields each player only his security level (i.e., his *maxmin* value, the amount he can guarantee for himself), but such that  $\sigma_1$  does not, in fact, guarantee the security level. Under these circumstances it is hard to see why the players would use their equilibrium strategies.<sup>1/</sup> Another striking example is that <sup>2/</sup> of Figure 2. Here there is a unique equilibrium point, yielding each player his *maxmin* payoff of 3. The equilibrium point, is, however, highly unstable; the slightest deviation leads to a cycle

Figure 2

0,0	4,5	5,4
5,4	0,0	4,5
4,5	5,4	0,0

that meanders among the strategy pairs yielding (4,5) and (5,4) -- both of which Pareto dominate the equilibrium payoff. A well known example is the 100-fold repetition of the prisoners' dilemma; its unique equilibrium point is for both players always to double cross.

Next, we turn to the core, also widely applied in economics. The simplest non-trivial example of a game whose core contains a unique point is the market with one buyer and two sellers (von Neumann and Morgenstern [1947], p. 564 ff.) with coalitional worth ("characteristic") function  $v(123) = v(12) = v(13) = 1$ ,  $v(S) = 0$  for all other  $S \subset \{1,2,3\}$ . The unique core point is (1,0,0); the buyer gets all the gains from trade, the sellers nothing. Here the Shapley value, (2/3, 1/6, 1/6), seems distinctly more reasonable. In another example (Aumann, Gardner, and Rosenthal [1977]), a continuum of workers can produce a public good that cannot be enjoyed by them, but only by a disjoint continuum of consumers; the consumers are incapable of producing anything (maybe they should be called drones or parasites). A side payment commodity is available to all agents. There is a unique point in the core, under which the workers produce all they can of the public good, and get paid nothing for it; this is also the unique

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Lindahl equilibrium. There are several other published examples of cores with strange, "counterintuitive" properties.<sup>3/</sup>

Another well known solution concept, the nucleolus of Schmeidler [1969], is subject to the "Alabama"<sup>4/</sup> paradox: the payoff to a player may go down when the worth  $v(N)$  of the all-player coalition goes up (Megiddo, [1974]).

Even the holy of holies of classic game theory, the minmax value of two-person zero-sum games, is not immune to counter-intuitive examples. Aumann and Maschler [1972] adduce an example in which the guaranteed value arguments that underlie the minmax concept lose their force, and one is left only with the equilibrium argument; and in this particular example, the latter is no more convincing than in the game of Figure 1.

The best known, most classical concepts of economic theory are also beset with counter-intuitive examples. We have already noted how strangely the Lindahl equilibrium may behave. But even the competitive equilibrium may misbehave. There are economies (Gale [1974]) in which one agent can give away part of his initial bundle to another agent, and gain in the competitive equilibrium by so doing. In a market for totally complementary goods--e.g. right and left gloves--in which there are initially 101 right gloves and 100 left gloves, the owners of the right gloves simply give their merchandise, for nothing, to the owners of the left gloves. This is well-known, textbook stuff; we have gotten used to it. But that doesn't make it any easier to swallow.

Perhaps the most apt examples come from the area of price indexation. "Apt", because the Shapley value--NTU as well as TU--is

perhaps best viewed as an index of a game for each of its players,<sup>5/</sup> a way of summing up a complex situation in a single number. No way of doing that is going to yield what we expected in every single instance. And indeed, there is no known method of price indexation that is not subject to pathologies and quirks; often these are present not only in artificially constructed examples, but in real data as well. That being the case, why should we expect the far more complex and general problem of "game indexation" to be entirely "quirk-free"?

2. But if solution concepts do not always yield intuitively satisfying results, how can we rely on them? What good are they?

To come to grips with this question, we must first back off a little and ask ourselves what it is we are trying to do in economic theory, or for that matter in science generally. On the most basic level, the answer is that we would like to understand our world. We sometimes test our comprehension with predictions, and often apply it in various ways; but the basic aim of scientific activity remains the comprehension itself.

Comprehension is a complex concept, with several components. One component has to do with fitting things together, relating them to each other. To "understand" an idea or a phenomenon--or even something like a piece of music--is to relate it to familiar ideas or experiences, to fit it into a framework in which one feels "at home". Unification is another component; the larger the framework--the more ground it covers--the better the understanding. Yet another component is simplicity,

sparseness; for example, with over 70 "elementary" particles at last count, we feel that our understanding of particle physics leaves something to be desired.

A solution concept may be viewed as a tool for analyzing and understanding a situation. To be useful, the outcome of such an analysis cannot stand by itself—it must be "explained in words," to be related to something different that lies outside of it. Perhaps the most spectacular applications of game theory are when you get a result that looks really crazy, and you say, "hey, what's going on here?", and you think and think about it, and suddenly something clicks, you see the situation in a completely new light, and you understand it.

Schumpeter once complained that economics is "halb selbstverstaendlich, halb unverstaendlich". The most interesting economics consists of the exceptions to Schumpeter's dictum; the 6 percent that while not obvious, and perhaps even puzzling, can be understood. Game Theory, including the NTU value, must be credited with a fair share of these insights.

Not all game theoretic insights are of this nature. In some cases, while the results themselves may be far from obvious, once they are obtained the interpretation is quite apparent. This is true, for example, of the results that relate the NTU values of a market to its competitive equilibria.<sup>6/</sup> Nevertheless, such results do increase our understanding by establishing new "angles" and new relationships, and by enabling a unified view of apparently disparate phenomena. I find it fascinating that the law of supply and demand in economics is related via the Shapley value to the disproportionate power of large coalitions

in certain electoral situations.<sup>1/</sup> Such results also cast light in the reverse direction--from the game and its solution onto the solution concept, thus making the solution concept more valuable in subsequent applications.

It sometimes also happens that through frequent use, a solution concept becomes so familiar that it itself becomes the explanatory framework. This has happened, for example, with the notion of competition. The one-point core  $\{(1,0,0)\}$  of the market with one buyer and two sellers seems strange and counterintuitive when viewed as an isolated phenomenon; but once we note that it is the competitive solution, we "understand" it. The same thing may in the future happen with any solution concept, including the NTU value; if so, we will "understand" examples like Roth's or Shafer's simply by noting that the payoff vector in question is in fact a value outcome. It's not a question of getting a formal stamp of approval from somewhere, but rather a process in which the mind ties together many applications and ideas until they jell into a coherent intuitive concept in their own right. Whether or not this happens with a particular solution concept depends on how much it is used and how successful it is in applications.

In brief, the proof of the pudding is in the eating of it.

A solution concept must be judged by its success in increasing our understanding of the situations to which it is applied. Like computational methods--e.g. Newton's method or the simplex method--it is a tool rather

than a dogma; and like all tools, it sometimes works and it sometimes doesn't. A tool that rarely works should be discarded; so should a clumsy tool, that is difficult to work with and yields mediocre results. But nobody would discard a generally fine tool just because he has been unsuccessful with it on one job. If your algorithm doesn't converge in real time, or if your solution method leads to bizarre results of which you can't make any sense, your tool hasn't worked for you; try another one. It would be good to understand why it hasn't worked, but it certainly doesn't mean you should throw out the tool.

3. One of the major advantages of the NTU value is its simplicity. This simplicity has two dimensions. First, the actual definition--in terms of the axioms for the TU Shapley value, and the extension to NTU games by means of the multipliers  $\lambda_i$  --is relatively uncluttered and easily grasped. The second, even more important dimension is ease of use. Mathematically, the value (TU or NTU, as the case may be) is perhaps the most tractable of all the concepts of cooperative game theory. In games with many players, there is even an informal calculus that often enables one to get quick answers, though rigorous proofs usually turn out to be more challenging.

It's easy to laugh off the matter of mathematical tractability. Of course, it can't be the only criterion; there's got to be some relationship between your method and the problem. In practice, though, it's more common to underestimate than to overestimate the importance of mathematical tractability. One hears endless discussion of conceptual minutiae in proposed new solution concepts, often leading to definitions

that are so cumbersome or complex that they are rarely or never applied to anything. While the definition of value may have some controversial aspects, it is not on the face of it absurd; and its simplicity and mathematical tractability has led to the growth of a considerable theory, which in turn has enabled a wide range of applications to Economics and Political Science. The payoff--the acid test--is in these applications. If the definition generally leads to reasonable results, and enables us to gain important new insights, then it's a good definition. If it leads to few or no results, it's not a good definition, no matter how reasonable it appears.

b. Finally, I don't find the specific examples of Roth and Shafer particularly compelling. Let's concentrate on a generic instance of Roth's example. There are 3 players, who must share 1. By himself, each player can get 0. If Players 1 and 3, or 2 and 3, form a coalition that excludes the remaining player, then they must divide  $2/3 - 1/3$ , with Player 3 getting  $2/3$ . If 1 and 2 form a coalition that excludes 3, they must divide  $1/2 - 1/2$ . If all three form a coalition, they may use a random device of their choosing to pick a 2-person coalition, which must then divide as above. Utilities are linear.

When the interpersonal comparison rates  $\lambda_i$  are restricted to be positive, the unique NTU value is  $(1/3, 1/3, 1/3)$ . But Roth argues that 3 is weak, because he can only offer 1 or 2 a payoff of  $1/3$ . Players 1 and 2 would therefore spurn 3's offers, and gravitate toward

each other. Roth concludes that the outcome must be  $(1/2, 1/2, 0)$ ; it is the "unique outcome ... consistent with the hypothesis that the players are rational utility maximizers ... the outcome  $(1/2, 1/2, 0)$  is strictly preferred by both players 1 and 2 to every other feasible outcome ... So ... there is really no conflict between players 1 and 2: their interests coincide in the choice of the outcome  $(1/2, 1/2, 0)$ , and the rules permit them to achieve this outcome." (Roth, [1980], pp. 458-9; his emphasis.)

Sounds logical; but let's look a little closer. Roth treats the situation as if it were a 2-person game, with only Players 1 and 2 involved. But it really is a 3-person game. Consider a bargaining model in which the players speak to each other in private, two at a time; if they agree, the bargain may be sealed immediately. Suppose 1 and 3 find themselves together. "A bird in the hand is worth two in the bush;" if 1 refuses to agree, he may find himself out in the cold, as 2 may agree with 3 at the next opportunity. The considerations of 2 are similar.

If one insists on viewing this as a 2-person game, then an appropriate matrix is that<sup>8/</sup> of Figure 3. In spite of Roth's argument,

Figure 3

$\frac{1}{2} , \frac{1}{2}$	$\frac{1}{4} , \frac{5}{12}$
$\frac{5}{12} , \frac{1}{4}$	$\frac{5}{18} , \frac{5}{18}$

I wouldn't consider  $(1/2, 1/2)$  a foregone conclusion at all. It will take guts for 1 and 2 to adopt the hold-out strategy; while potentially less profitable, it is safer to accept the first offer. Roth's simultaneous two-person "rationality" goes considerably beyond simple individual utility maximization. You've got to depend on the other person's "rationality," which in turn is indeed justified only if he depends on yours, and so on. Maybe; but I'm not entirely convinced.<sup>9/</sup>

To bring this into sharper focus, substitute  $(1/2 + \epsilon, 1/2 - \epsilon)$  for  $(2/3, 1/3)$ ; the matrix of Figure 3 is then replaced by that of Figure 4. The full force of Roth's argument continues to apply;

Figure 4

$\frac{1}{2}$ , $\frac{1}{2}$	$\frac{1}{4}$ , $\frac{1-\epsilon}{2}$
$\frac{1-\epsilon}{2}$ , $\frac{1}{4}$	$\frac{1-\epsilon}{3}$ , $\frac{1-\epsilon}{3}$

apparently, he would predict that "rational utility maximizers" would reach  $(1/2, 1/2, 0)$  (or  $(1/2, 1/2)$  in the two-person version) even then. For small  $\epsilon$ , this seems an unlikely proposition. We are, in fact, within  $\epsilon$  of a prisoner's dilemma; each of Players 1 and 2 has much to gain, and little to lose, by accepting the first offer.<sup>10/</sup>

In an even more compelling example, there are 10 players rather than 3; each of Players 1 through 9 can form a two-person coalition with Player 10, which must split .9 - .1 (in favor of 10); and the only

other coalition that can get anything consists of all the Players 1 through 9, which must split evenly. As before, the all-player coalition has the option of choosing a smaller coalition by a random device.

Again, the full force of Roth's reasoning applies; in principle, there is no reason that his kind of "rationality" should apply any the less to 9 --or for that matter 99 --people than to 2. So presumably, he would predict with certainty that Players 1 through 9 will form a coalition and split evenly; in the role of Player 1, he would reject any overtures from 10 with dignity but firmness. Perhaps I am irrational, but for his sake, I hope I am not Player 2; for he can be assured that I would accept any offers from 10 with alacrity, while he is out there trying to round up the other fellows.

All in all, Roth appears to have overstated his case by a good bit; there seems to be no universally valid general principle that leads inexorably to a coalition between 1 and 2 in the 3-person game with which we started this section. But let's not hold that against him; let's continue to examine the case on its merits. Granting that  $(1/2, 1/2, 0)$  is not a foregone conclusion, that perhaps 3 should get something in the value, isn't he still much weaker than the other players, and shouldn't that be reflected in the value?

Not necessarily. On the contrary, at first glance it would seem that he is stronger. After all, if he does get into a coalition, he will get more than any other player; so if all coalitions were equally likely, he should get more. Of course, all coalitions are not equally

likely, and that's what brings down his value. It's not that clear, though, which effect is stronger here.

To fix ideas, let's think of a democracy with 3 parties, holding 26%, 26%, and 48% of the seats in parliament respectively. A majority is required to form a government. There is a custom, which has the force of law, that cabinet posts within the government are divided as closely as possible in proportion to the representation in Parliament. The utility of each party is proportional to the number of cabinet posts it holds.

With 6 cabinet posts, this situation is precisely the game we have been considering. Roth suggests that the two smaller parties would necessarily form the government, that the large party is not only weaker than the small parties, but is actually completely powerless. But many people would say the opposite; that the large party has more power than the smaller parties. All in all, perhaps  $(1/3, 1/3, 1/3)$  is not totally unreasonable after all.

Finally, a few words about Shafer's examples. Mathematically, these are similar to Roth's, though smoother. The important difference is that they are set in a market context that enhances their credibility. Player 3's initial bundle is small; his high payoff if he forms a coalition with 1 or 2 stems from the form of his utility function. The initial bundle is more "visible" than the utility, and tends to drive Players 1 and 2 together.

Shafer may well have a point. Game Theory, as well as economics, typically provides multiple solutions. But game theoretic concepts

apply to a "purified" or "processed" version of the original situation, such as the coalitional<sup>11/</sup> or strategic<sup>12/</sup> form. The processing removes some of the "glue" that gives the situation coherence; to choose among the multiple solutions, it may be necessary to restore some of this glue, to go back and look at the "raw", original situation. This is related to the "focal points" discussed by Schelling [1960]; equilibrium points that are indistinguishable in the strategic form may well be distinguishable in the original situation, and there may be good reasons to prefer one over another.

The coalitional game under discussion here may be realized in the political context discussed above, or in the market context discussed by Shafer. In the political context, it appears more likely that a coalition including Player 3 will form; in the market context, it appears less likely. Other political and/or economic contexts might lead to other conclusions.

The Shapley value is "context-neutral." Based on the coalitional worth function only, it cannot take into account the peculiar features of each realization of this function. Thus it is best viewed as an average of outcomes that might be expected in the various specific contexts that realize the same coalitional worth function; and it is not surprising that it is at odds with what one might expect in a particular one of these specific contexts.

5. In conclusion, the work of Roth and Shafer has been important because it cast light on the nature and workings of the NTU value, and because it led to a better understanding of the particular situations they were addressing. But a solution concept must be judged primarily by how successful it is in increasing our understanding of economics and politics and of interactive phenomena in general. Included in the references is a partial bibliography of the NTU value, including a number of applications of this kind.<sup>13/</sup>

Notes

- 1/ The equilibrium and maxmin strategies are mixed, but that is not an issue; an example in which these phenomena occur in pure strategies may be constructed, simply by explicitly adding rows and columns that contain the payoffs of the appropriate mixed strategies.
- 2/ As far as I know, this game first appeared in print in Moulin and Vial [1978], in connection with correlated equilibria. Previous to that it had, however, been part of the folklore of game theory for a considerable time, as an example of the pathology of equilibria. In that context, I believe it is due to Lloyd Shapley.
- 3/ For example, Kalai, Postlewaite, and Roberts [1978].
- 4/ See Balinski and Young [1979].
- 5/ I owe this insight to Hugo Sonnenschein.
- 6/ Shapley and Shubik [1969], Champsaur [1975], Aumann [1975], Brown and Loeb [1976], Mas-Colell [1977], Hart [1977], Cheng [1981]. Some of these results, e.g., those of Hart, relate the value to specific kinds of competitive equilibria, and thus provide "new insights" as well as "new relationships".
- 7/ See, e.g., Riker and Shapley [1968], Shapiro and Shapley [1978], Milnor and Shapley [1978].
- 8/ The top (left) strategy of 1(2) is "hold out for 1/2"; the other strategy is "accept first offer." The payoffs are calculated on the basis of a sequence of two-person meetings, each one selected independently with probability 1/3, and held without the knowledge of the excluded player.
- 9/ It might be argued that we're talking about cooperative games, where people can make binding agreements; in such an environment, surely 1 and 2 can profitably and safely agree to play (Top, Left) even in the game of Figure 4. But the cooperative game is the three-person game--that's where the binding agreements can be made. If one insists on viewing this as a game between 1 and 2, then the strategic question at issue is whether they will agree with each other, or whether one of them will agree with 3. It's a mistake to talk about binding agreements in this preliminary game. Thus one must view Figure 3 as a non-cooperative game, where binding agreements are ruled out.

- 10/ If  $\epsilon \geq 1/4$ , the "hold out" strategy is dominant. One must remember, though, that this two-person game represents only one way of modelling the negotiating process; other ways of looking at the problem might well lead to different results. In any case, at this point we only wish to challenge Roth's basic reasoning, which led him to conclude in absolute terms that "rational" players would necessarily arrive at  $(1/2, 1/2, 0)$  (see text); and this reasoning is independent of  $\epsilon$ .
- 11/ "characteristic function".
- 12/ "normal".
- 13/ Items in this bibliography have not been separated in the references from other items referred to in the text. This bibliography is limited in scope to the NTU value as defined in Shapley [1969]; there have been several variants, notably those due to Harsanyi (see, e.g., Harsanyi [1977]), which we have not attempted to cover. Also, we have made no attempt to cover the very extensive literature of the TU value. Even with these restrictions, the bibliography makes no serious attempt at completeness, but the author would be grateful if he would be apprised of other items fitting the above description. Because the field is in a state of rapid development, several unpublished "preprints" have been included. The author is indebted to A. Mas-Colell for permission to use a bibliography compiled in connection with a one-day workshop that the latter gave on NTU values at the IMSSS summer seminar at Stanford in August of 1980.

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